1.
(a) The x in ∃y(¬p(x,y)) is free.
(b) Don’t always need true.
(c) B ⊆ A ∪ C
(d) Don’t always need true.
(e) (p ∨ q) ∧ (¬p ∨ q) ∧ (¬q ∨ r) ⇒ r
   (p ∨ q) ∧ ((p ∨ q) ⇒ r) ⇒ r
   (p ∨ q) ∧ (¬(p ∨ q) ∨ r) ⇒ r
   ((p ∨ q) ∧ (¬(p ∨ q)) ∨ (p ∨ q) ∧ r) ⇒ r
   F ∨ ((p ∨ q) ∧ r) ⇒ r
   (p ∨ q) ∧ r ⇒ r
   ¬((p ∨ q) ∧ r) ∨ r
   (¬(p ∨ q) ∨ ¬r) ∨ r
   (¬(p ∨ q)) ∨ (¬r ∨ r) ≡ T
4. (10%) 
\[ \neg q \text{ (Assumption)} \quad \neg p \rightarrow r \]
\[ p \rightarrow q \quad (\text{MT}) \quad r \rightarrow s \quad (\text{HS}) \]
\[ \neg p \quad s \quad (\text{MP}) \]
\[ \neg q \rightarrow s \]

5. (15%) 
(a) (6%) 
(i) F  
(ii) T  
(iii) F  
(iv) F  
(v) T  
(vi) F  
(b) (4%) 
False.  
If \( P(x) : x \neq a \)  
\( Q(x) : x \neq b, \) and \( a \neq b \)  
then the first proposition is true, but the second proposition is false.  
(c) (6%) 
No.  
If \( P(x) : x = a, \) then \( \neg \exists ! x P(x) \) is false but \( \forall x \neg P(x) \) is true. \( \exists ! x P(x) \equiv \forall x \neg x(\neg P(x)) \)

6. (15%) 
(a) (5%) 
\[ LI \equiv \left( poly = \sum_{k=0}^{i-1} x^k \right) \land \left( i \leq n + 1 \right) \]
(b) (10%) 
(i) Prove \( LI \) is a loop invariant.  
Suppose \( LI \) is true and the condition of the \texttt{while} loop holds, i.e.,  
\[ poly = \sum_{k=0}^{i-1} x^k \text{ and } i \leq n . \]
Then, the new values become \( i_{\text{new}} = i + 1 \) and
\[ poly_{\text{new}} = poly \times x + 1 = \sum_{k=0}^{i-1} x^k \times x + 1 = \sum_{k=0}^{i} x^k = \sum_{k=0}^{i_{\text{new}}-1} x^k. \]

Since \( i \leq n \), we have \( i_{\text{new}} \leq n+1 \).

Thus \( LI \) is true after the execution of \textbf{while} loop.

ii) Consider the program segment.

Both \( i = 1 \) and \( poly = 1 = \sum_{k=0}^{i-1} x^k = \sum_{k=0}^{i-1} x^k \) hold before entering the loop, so \( LI \) is true.

Since \( LI \) is a loop invariant, if the while loop terminates, it terminates with \( LI \) true and with \( i \leq n \) false.

In this case, it suggests \( i = n+1 \) and

\[ poly = \sum_{k=0}^{i-1} x^k = \sum_{k=0}^{n+1-1} x^k = 1 + x + x^2 + \ldots + x^n. \]

iii) Check the \textbf{while} loop terminates.

i gets the initial value 1, after \( n \) iterations, the new value of i will be \( n+1 \).

And the loop terminates at that point.

7. (20%)
(a) \textbf{Well-ordering property} of natural numbers. (4%)

Every nonempty set \( S \) of natural numbers contains a least element; that is, there is some number \( a \) in \( S \) such that \( a \leq b \) for all \( b \) belonging to \( S \).

(b) The \textbf{continuum hypothesis} (4%)

The Continuum Hypothesis states that there is no cardinal number \( \kappa \) such that \( \aleph_0 < \kappa < 2^{\aleph_0} \).

The continuum hypothesis can also be stated as: there is no subset of the real numbers which has cardinality strictly between that of the reals and that of the integers. It is from this that the name comes, since the set of real numbers is also known as the continuum.

(c) Two infinite sets having the same \textbf{Cardinality} (4%)

\textit{Cardinality} is a notion of the size of a set which does not rely on numbers. It is a relative notion. For instance, two sets may each have an infinite number of elements, but one may have a greater cardinality. That is, in a sense, one may have a “more infinite” number of elements.
The cardinality of a set $A$ is the unique cardinal number $\kappa$ such that $A$ is equal-numerous with $\kappa$. The cardinality of $A$ is written $|A|$.

(d) **Countable set** (4%)
A set $S$ is countable if there exists a bijection between $S$ and some subset of $\mathbb{N}$.

(e) **Partial correctness** in program verification (4%)
The correct answer is obtained if the program terminates.
A program $S$ is said to be partially correct with respect to the initial assertion $p$ and the final assertion $q$ if whenever $p$ is true for the input values of $S$ and $S$ terminates, then $q$ is true for the output values of $S$. 