1. (10 pts) True or False? Score = max{0, right - \frac{1}{2} wrong}. No explanations are needed.

  (a) Relation \( R = \{(1,1), (1,2)\} \) (over the set \{1,2\}) is an antisymmetric relation.
  (b) A partial order relation is a relation which is reflexive, asymmetric and transitive.
  (c) If a poset has a least element, then the element is unique.
  (d) If \((S, R)\) is a poset, then \((S, R^{-1})\) is also a poset. (Here \(R^{-1}\) denotes the inverse of \(R\).)
  (e) There is a simple graph with 4 vertices each having degrees 2, 2, 3, 3.
  (f) Both \(K_9\) and \(K_{10}\) have an Euler cycle.
  (g) Graph \(K_{5,4}\) has 20 edges.
  (h) Let \(G\) be a simple graph with 5 vertices and 9 edges. Then \(G\) is always planar.
  (i) Every loop-free connected planar graph has a vertex \(v\) with degree\((v) < 6\).
  (j) Let \(R\) be the relation on the set of all ordered pairs of positive integers such that \((a, b)R(c, d)\) if and only if \(ad = bc\). The \(R\) is an equivalence relation.

2. (10 pts) Define Ramsey\((g, y)\) to be the smallest number \(n\) such that any green/yellow coloring of the edges of an \(n\)-clique will contain a green \(g\)-clique or a yellow \(y\)-clique, where \(g, y\) are natural numbers. Use an example to show that \(Ramsey(3, 3) > 5\). That is, construct a graph with two colors (green and yellow) such that the graph neither has a green 3-clique nor has a yellow 3-clique.

3. (15 pts) If \(A = \{1, 2, 3, 4\}\) and \(R = \{(1, 2), (1, 3), (2, 4), (3, 2)\}\) be a binary relation over \(A\). Answer the following questions. No explanations are needed. (Recall that \(R^2 = R \circ R\).)

  (a) \(R^2 = ?\)
  (b) \(R^3 = ?\)
  (c) \(R^4 = ?\)
  (d) What is the symmetric closure of \(R\)?
  (e) What is the transitive closure of \(R\)?

4. (10 pts) Let \(A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}\), and define relation \(R\) on \(A\) by \((x_1, y_1)R(x_2, y_2)\) if \(x_1 + y_1 = x_2 + y_2\).

  (a) Prove that \(R\) is an equivalence relation.
  (b) Determine the equivalence classes \([1, 3]\) and \([1, 1]\) (i.e., for each of the two equivalence classes, list all of its elements).
5. (10 pts) Solve the following recurrence relation exactly. Show your derivation in detail.

\[ a_{n+2} - 4a_{n+1} + 3a_n = -200, \quad n \geq 0, \]
\[ a_0 = 3000, \quad a_1 = 3300. \]

6. (15 pts) Consider the following graph \( G \)

(a) (10 pts) Compute the the chromatic polynomial \( P(G, \lambda) \). Show your derivation in detail.

(b) (5 pts) Find the chromatic number \( \chi(G) \).

7. (10 pts) Prove (in a rigorous way) that the following graph does not have a Hamilton circuit.

8. (20 pts) Answer the following questions:

(a) Construct a graph \( G_1 \) which has a Hamilton circuit but not a Euler circuit
(b) Construct a graph \( G_2 \) which has a Euler circuit but not a Hamilton circuit
(c) Construct a graph \( G_3 \) which has both a Hamilton circuit and a Euler circuit
(d) Construct a graph \( G_4 \) which is not planar but has a Hamilton circuit.
(e) Construct a graph \( G_5 \) which is homeomorphic (but not isomorphic) to \( K_{3,3} \).