Probability and Statistics

Quiz One Solution

Statement A: False

Sol :

Let $A = C$.

Events A and B are independent and events B and C are in dependent.

Obviously, events A and C are not independent.

Statement B: False

Sol :

Let B be an event with $P[B] = 1$.

$\because$ Events A and B are indepedent.

$\therefore P[A \cap B] = P[A] \cdot P[B] = P[A] \cdot 1 = P[A]$

$\Rightarrow$ A is a subset of B.

Statement C: False

Sol :

Let $P[A] > 0$ and $P[B] > 0$.

If Events A and B are mutually exclusive, then $P[A \cap B] = 0 \neq P[A] \cdot P[B]$.

$\Rightarrow$ Events A and B are not independent.

Statement D: True

Sol :

Let $B = \{X \le x\}$, $C = \{X > x\}$, S : universal set.

$\because P[B \cap C] = 0,\ P[B \cup C] = P[S] = 1$

$$
\begin{aligned}
\therefore P[A] &= P[A \cap S] \\
&= P[A \cap (B \cup C)] \\
&= P[(A \cap B) \cup (A \cap C)] \\
&= P[A \cap B] + P[A \cap C] \\
&= \frac{P[A \cap B]}{P[B]} P[B] + \frac{P[A \cap C]}{P[C]} P[C] \\
&= P[A \mid B] P[B] + P[A \mid C] P[C] \\
&= P[A \mid X \le x] P[X \le x] + P[A \mid X > x] P[X > x] \\
&= P[A \mid X \le x] F_X(x) + P[A \mid X > x](1 - F_X(x))
\end{aligned}
$$

Statement E: True

Sol :

The pdf of X is represented by $f_X(x)$.

$\because f_X(x)$ is an even function.

$\therefore F_X(-x) = \int_{-\infty}^{-x} f_X(t)dt = \int_{x}^{\infty} f_X(t)dt = 1 - \int_{-\infty}^{x} f_X(t)dt = 1 - F_X(x)$

$\because F_X(x_u) = u$

$\therefore F_X(x_{1-u}) = 1 - u = 1 - F_X(x_u) = F_X(-x_u)$

$\because F_X(x)$ increases monotonically with its argument x

$\therefore x_{1-u} = -x_u$